COMMTECH knowledge

RF, Microwave & Wireless

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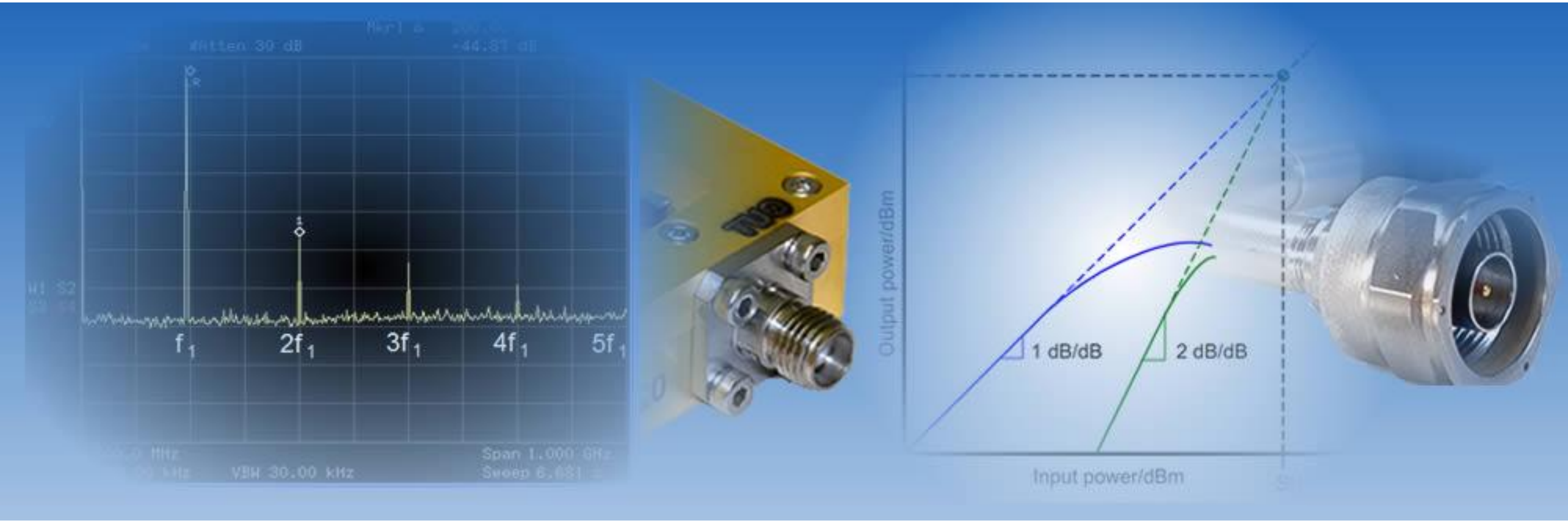


Non-Linearity Phenomenon

Physical causes of nonlinearity

- Operation under finite power-supply voltages
- Essential non-linear characteristics of electronic active components (transistors, diodes, etc.)
- Mismatch of input signal levels to a design
- Mismatch of number of input signals to a design

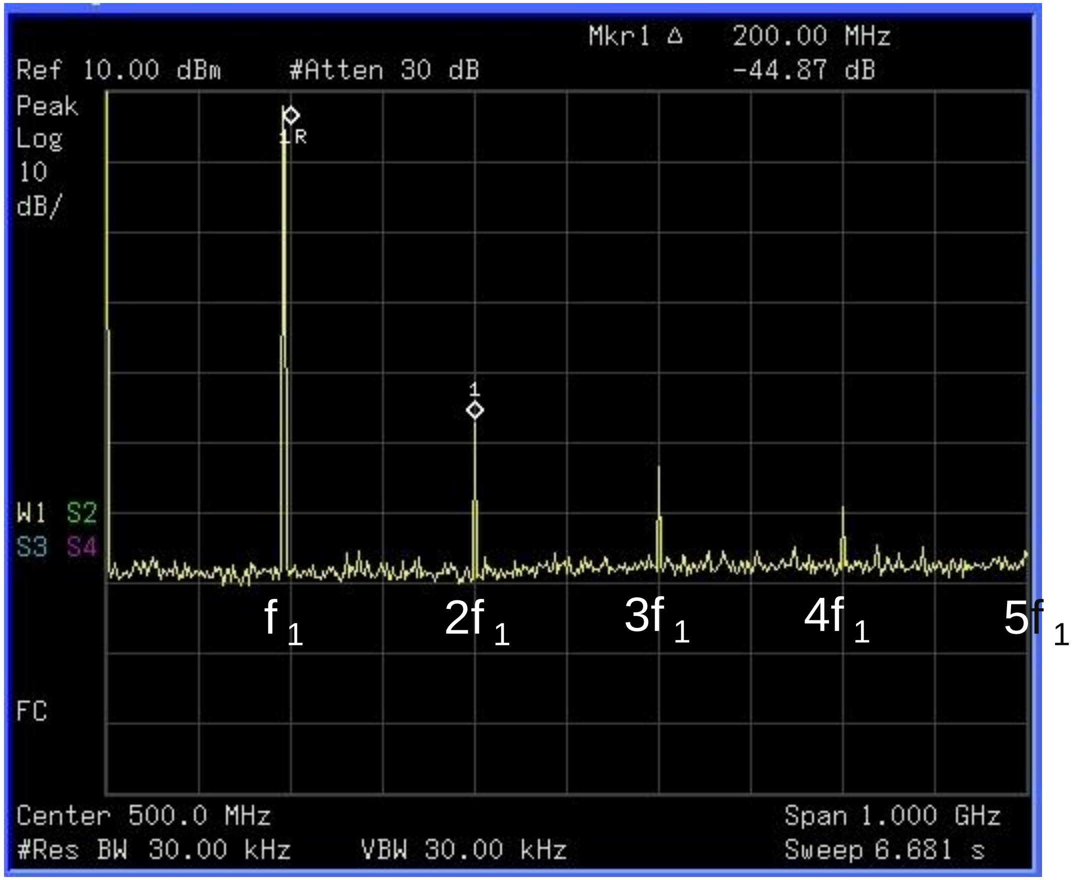
Problems caused by nonlinear distortions

- Transmission
  - Harmonics
  - Emission Mask spillover
  - EVM and Image Rejection degradation
  - Reduce efficiency (by backoff)

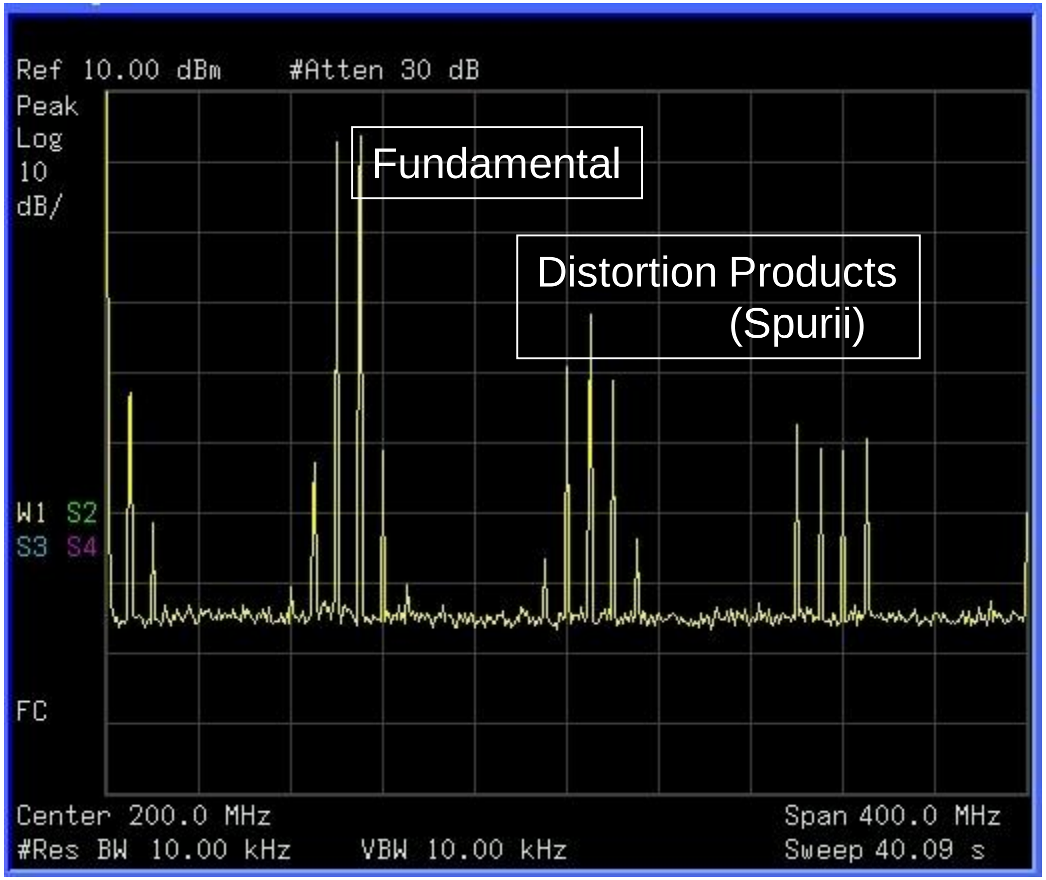
Problems caused by nonlinear distortions

- Reception
  - Spurii (“signals” show up, even if nonexistent at input)
  - Reduce dynamic range
  - Reduce sensitivity (desensitization)
  - Blocking of desired signals

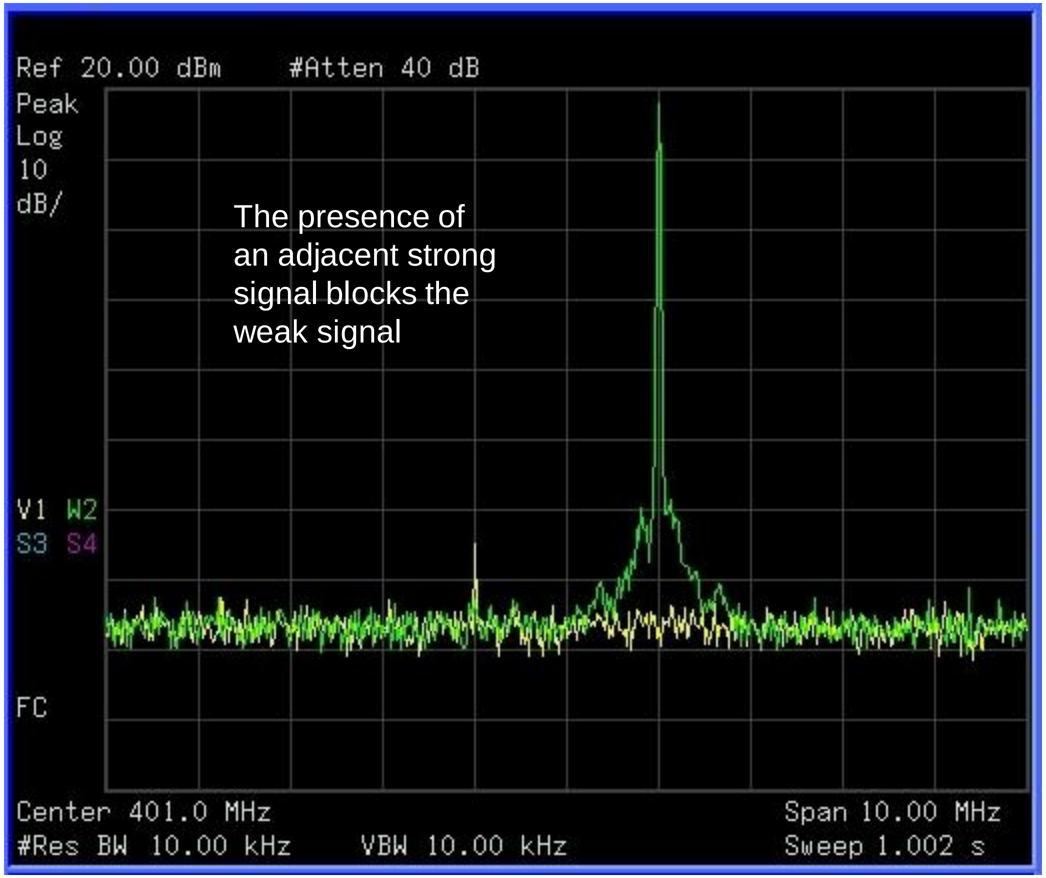
Harmonic Distortion



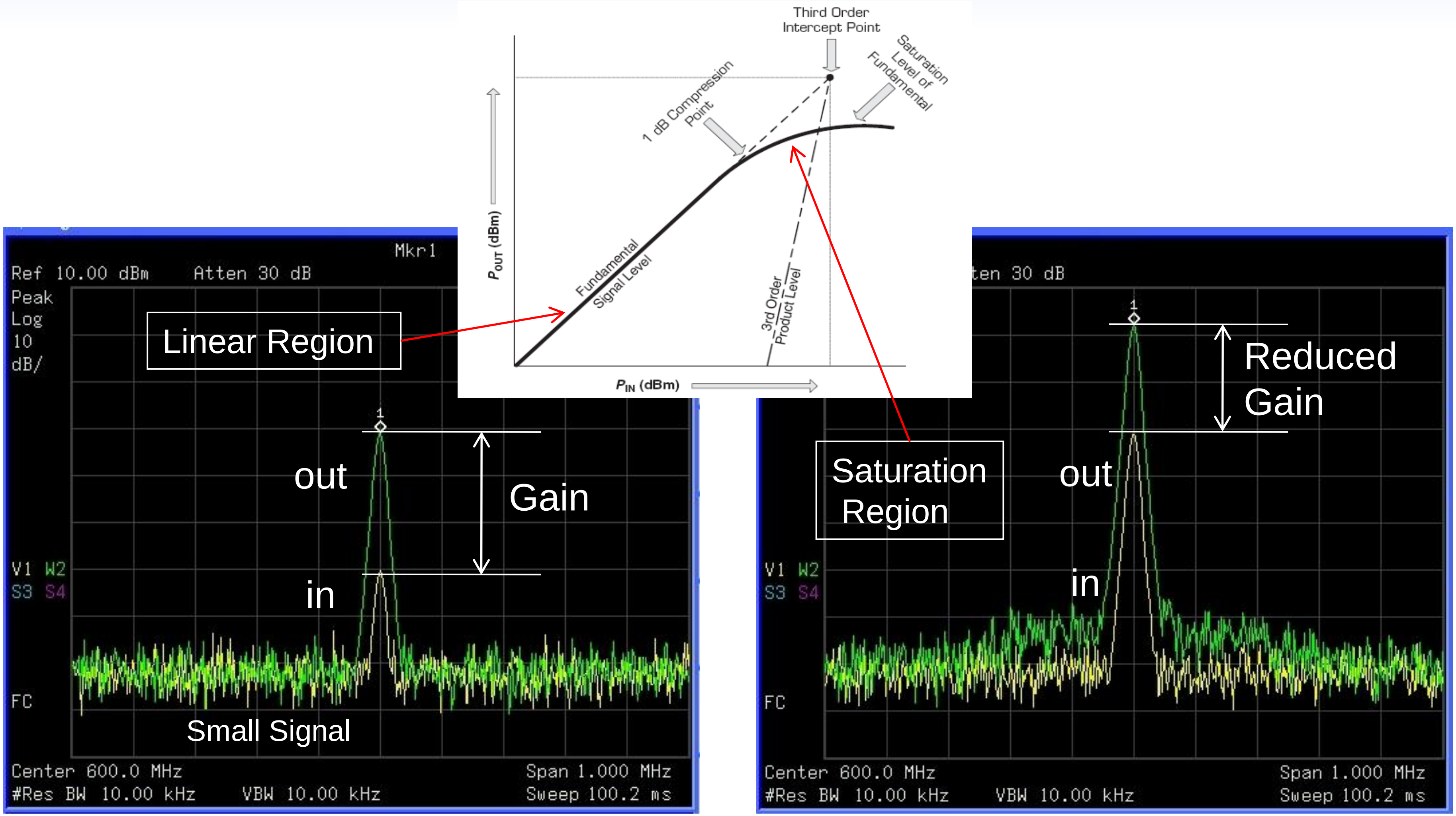
Intermodulation



Blocking (De-Sensing)



Compression



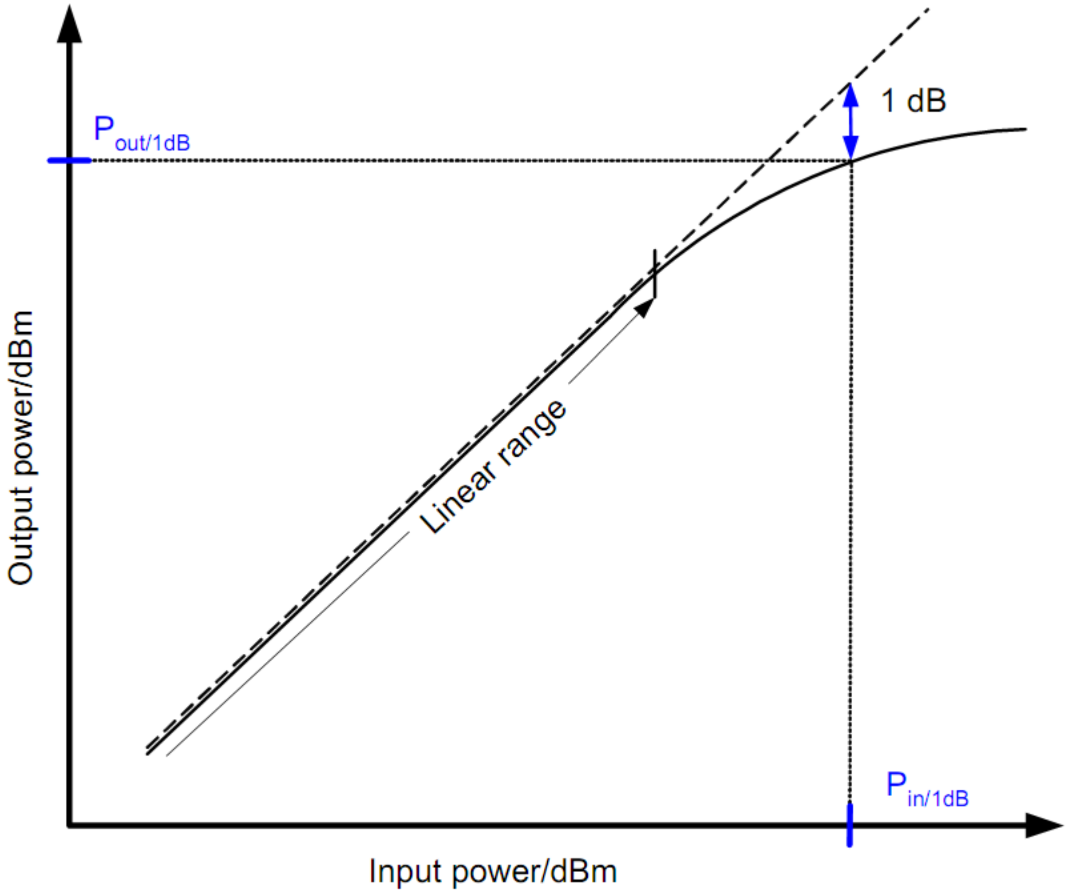
1 dB compression point

- Definition

The 1 dB compression point specifies the output power of an amplifier at which the output signal lags behind the nominal output power by 1 dB.

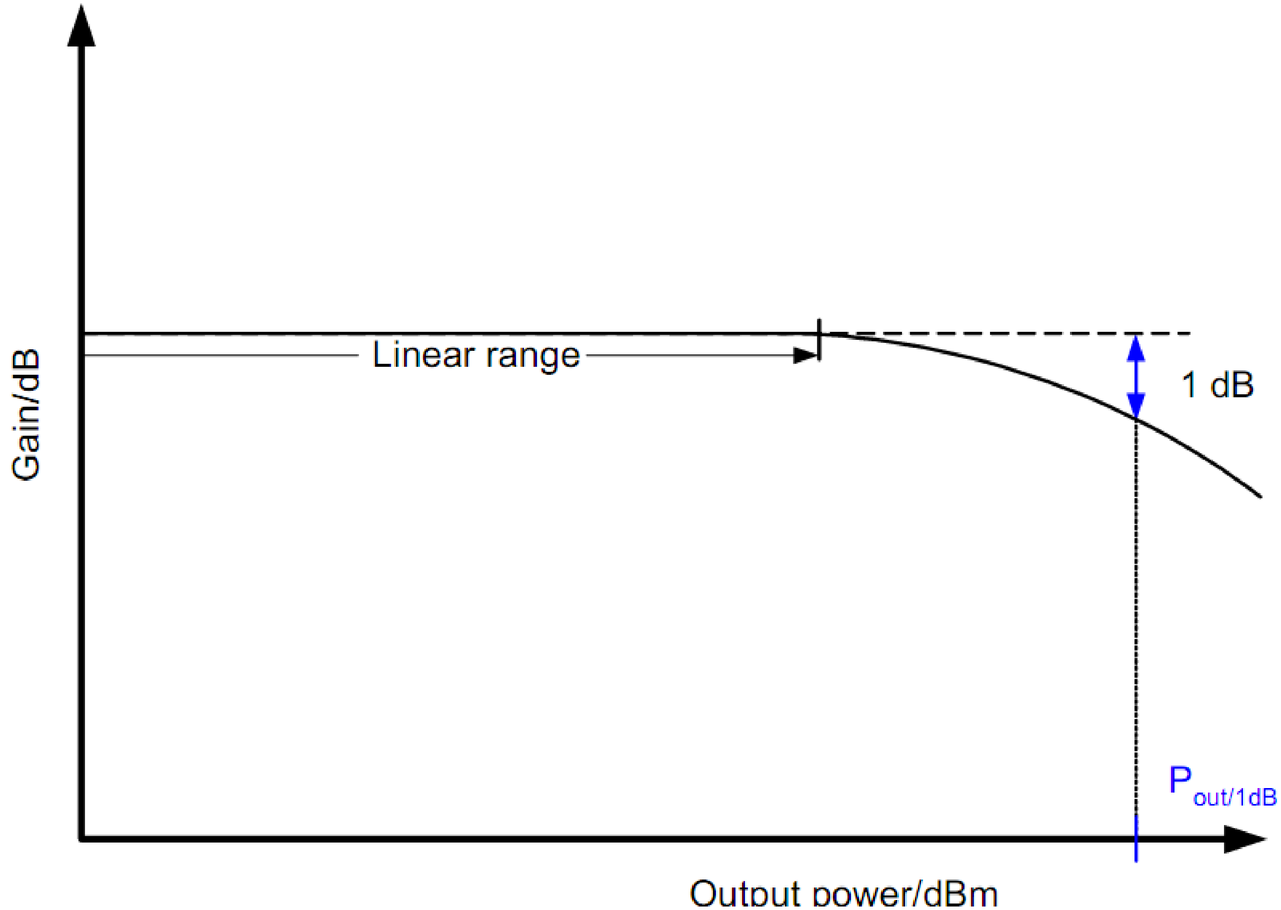
Compression

- Definition of the 1 dB compression point at the amplifier input (Pin/1dB) and at the amplifier output (Pout/1dB)

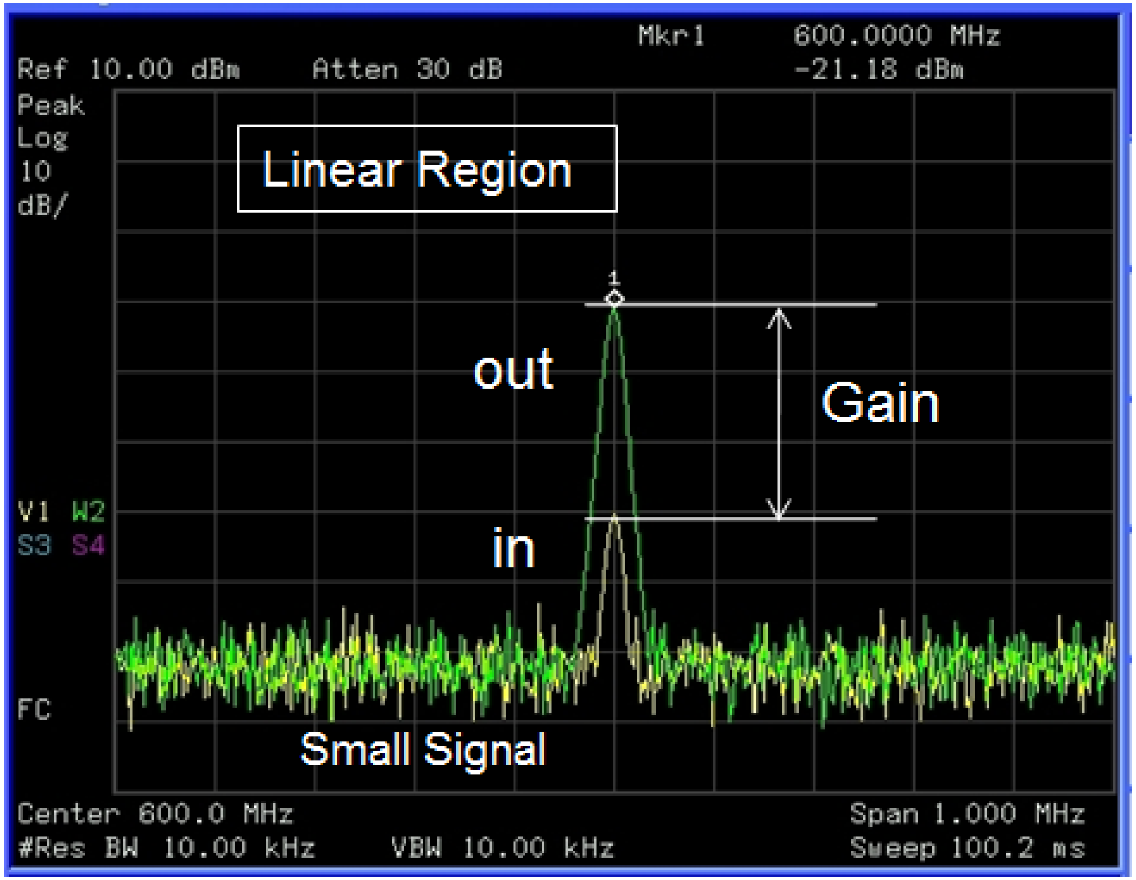


Compression

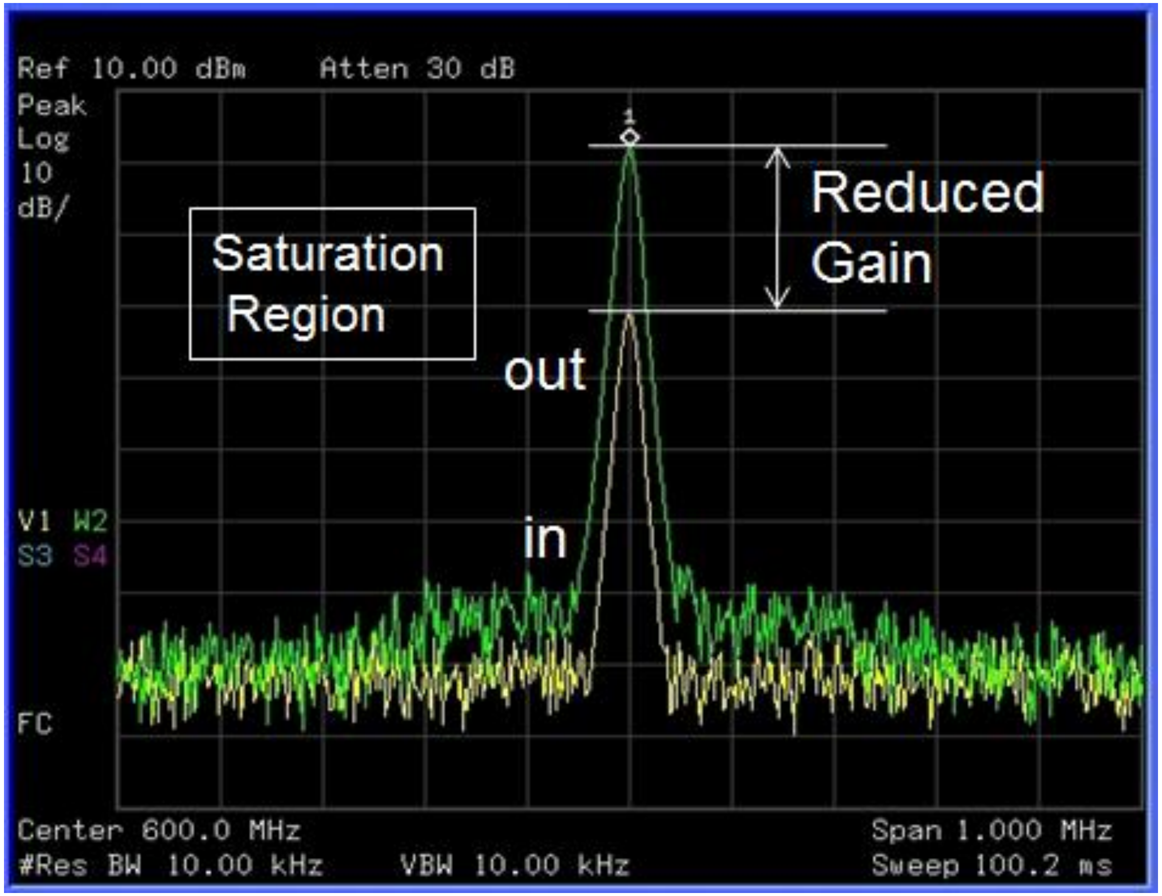
- Gain versus output power and definition of the 1 dB compression



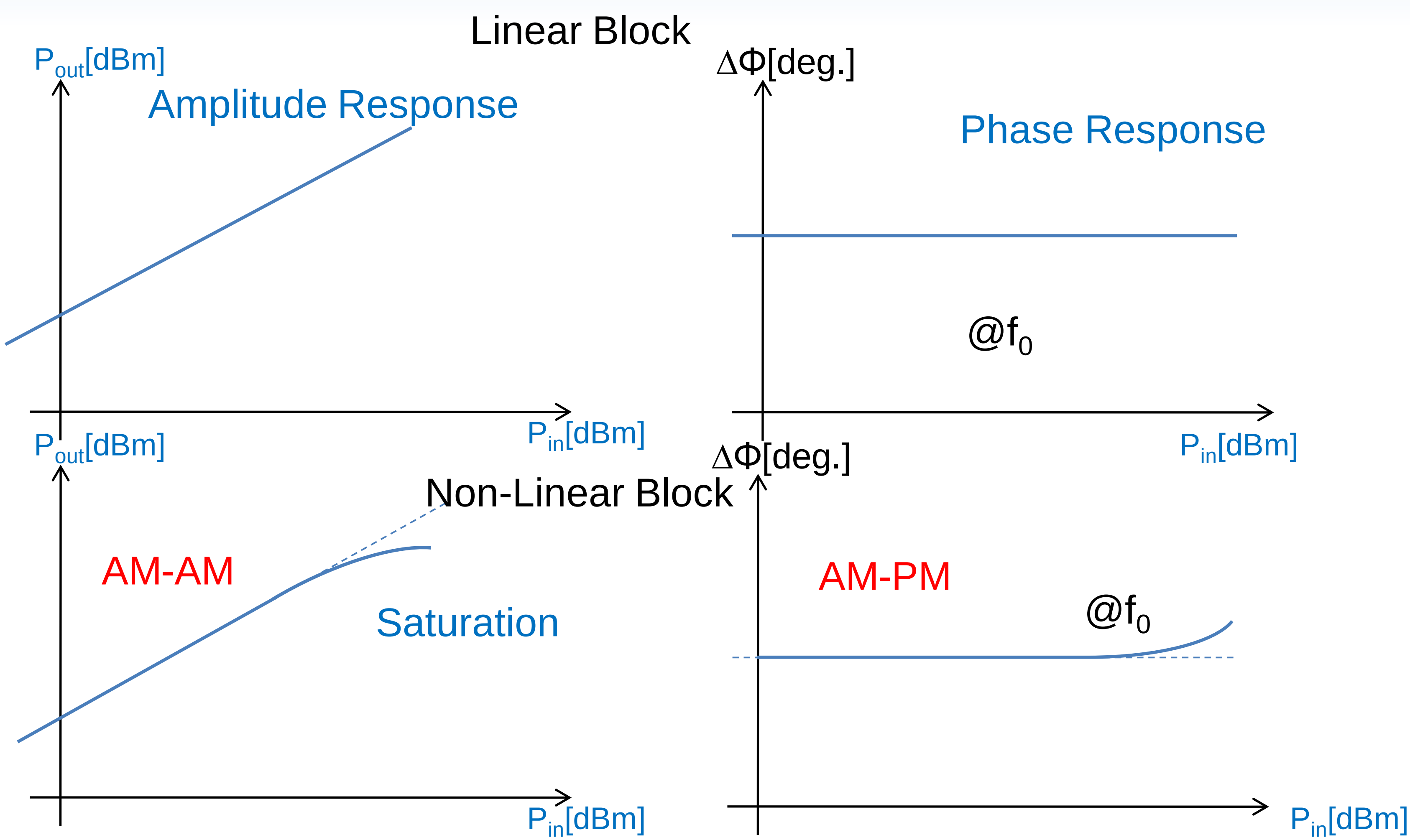
Linear Region



Saturation Region



Models of nonlinear blocks and their characterization



Nonlinearities

- An ideal amplifier

$$P_{out}(t) = G_P \cdot P_{in}(t)$$

where $P_{out}(t)$ power at output of twoport
$P_{in}(t)$ power at input of twoport
G_P power gain of twoport

The connection to the input and output voltage is as follows:

$$P_{in}(t) = \frac{1}{R_{in}} \cdot v_{in}^2(t)$$

$$P_{out}(t) = \frac{1}{R_L} \cdot v_{out}^2(t)$$

The voltage transfer function of the linear two-port is as follows:

$$v_{out}(t) = G_v \cdot v_{in}(t)$$

Nonlinearities

- In practice

$$v_{out}(t) = \sum_{n=0}^{\infty} a_n \cdot v_{in}^n(t) = a_0 + a_1 \cdot v_{in}(t) + a_2 \cdot v_{in}^2(t) + a_3 \cdot v_{in}^3(t) + \ldots$$

where $v_{out}(t)$ voltage at output of two-port
$v_{in}(t)$ voltage at input of two port
a_0 DC component
a_1 gain $\sqrt{G}$
a_n coefficients of the nonlinear element of the voltage gain

Single-tone driving – Harmonics

► If a single sinusoidal signal $v_{in}(t)$ is applied to the input of the two port

$$v_{in.}(t) = \hat{V}_{in}.\sin(2\pi f_{in,1} \cdot t)$$

$\hat{V}_{in}$: peak value of $v_{in}(t)$

and

$f_{in,1}$: frequency of $v_{in}(t)$,

$$v_{in.}(t) = \hat{V}_{in}.\sin(\omega_{in,1} \cdot t)$$

$\omega_{in,1}(t) = 2\pi f_{in,1}$ (angular frequency)

this is referred to as single-tone driving.

$$v_{out}(t) = a_0 + a_1 \cdot v_{in}(t) + a_2 \cdot v_{in}^2(t) + a_3 \cdot v_{in}^3(t) + \ldots. =$$

Single-tone driving – Harmonics

- Applying the trigonometric identity:

$$\sin^2(x) = \frac{1}{2}(1 - \cos 2x) \text{ and } \sin^3(x) = \frac{1}{4}(3 \cdot \sin x - \sin 3x)$$

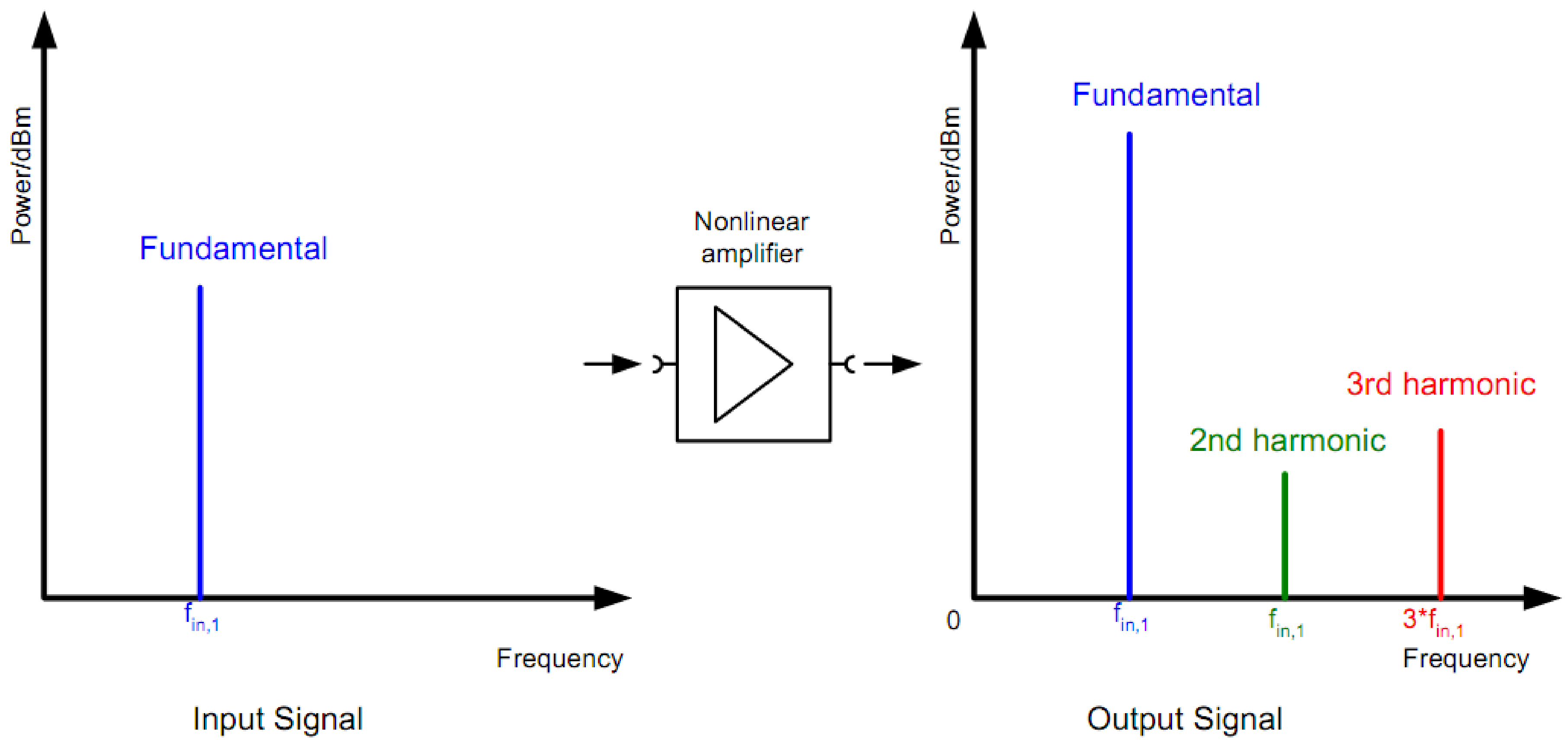
$$a_0 + a_1 \cdot \hat{V}_{in}(t)\sin\ (\omega_{in,1} \cdot t) + a_2 \cdot \hat{V}_{in}^2 \cdot \sin^2(\omega_{in,1} \cdot t) + a_3 \cdot \hat{V}_{in}^3 \cdot \sin^3(\omega_{in,1} \cdot t) + ...$$

$$= a_0 + a_1 \cdot \hat{V}_{in}(t) \cdot \sin\ (\omega_{in,1} t) + 0.5 \cdot a_2 \cdot \hat{V}_{in}^2 - 0.5 \cdot a_2 \cdot \hat{V}_{in}^2 \cdot \cos(2\omega_{in,1} t) +$$
$$+ 0.75 \cdot a_3 \cdot \hat{V}_{in}^3 \cdot \sin \omega_{in,1} t - 0.25 \cdot a_3 \cdot \hat{V}_{in}^3 \cdot \sin(3\omega_{in,1} t)......$$

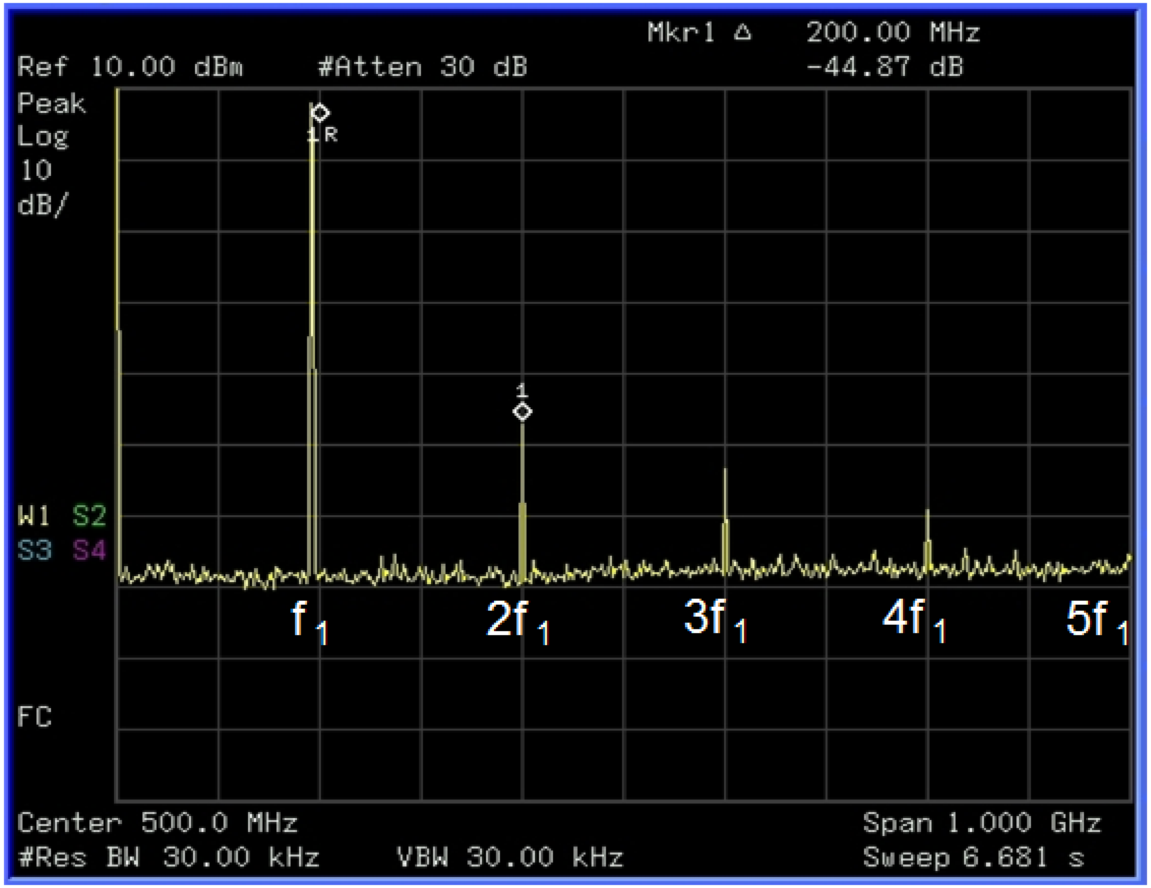
$$= a_0 + 0.5 \cdot a_2 \cdot \hat{V}_{in}^2 + (a_1 \cdot \hat{V}_{in} + 0.75 \cdot a_3 \cdot \hat{V}_{in}^3) \cdot \sin\ (\omega_{in,1} t) +$$
$$- 0.5 \cdot a_2 \cdot \hat{V}_{in}^2 \cdot \cos(2\omega_{in,1} t) - 0.25 \cdot a_3 \cdot \hat{V}_{in}^3 \cdot \sin(3\omega_{in,1} t)......$$

Single-tone driving – Harmonics

- Spectrum before and after a nonlinear two-port block:



Single-tone driving – Harmonics



Single-tone driving – Harmonics

- The levels of harmonics increase over proportionally with their order as the input level increases, i.e.
  - Changing the input level by A dB
  - Changes the n^{th} harmonic level by $n \cdot A$ dB

Note: This assumes the memory-less modelling applies.

Two-tone driving – Intermodulation

- Two-tone driving applies a signal v(t) into the input of the two-port block.
- This signal consists of the sum of two sinusoidal harmonic tones.

$$V_{in.}(t) = \hat{V}_{in,1} \cdot \sin(2\pi f_{in,1} \cdot t) + \hat{V}_{in,2} \cdot \sin(2\pi f_{in,2} \cdot t)$$

where $\hat{V}_{in,1,2}$ peak values of the two sinusoidal signals

$f_{in,1}$, $f_{in,2}$ signal frequencies

$\omega_1 = 2 \cdot \pi \cdot f_{in,1}$ and $\omega_2 = 2 \cdot \pi \cdot f_{in,2}$.

Two-tone driving – Intermodulation

- The new frequencies produced may be evaluated using the following trigonometric identities:

$$\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$$

$$\sin^2(x)\cdot\sin(y) = \frac{1}{2}(1 - \cos 2x)\cdot\sin(y)$$

$$\sin^3(x) = \frac{1}{4}(3\cdot\sin x - \sin 3x)$$

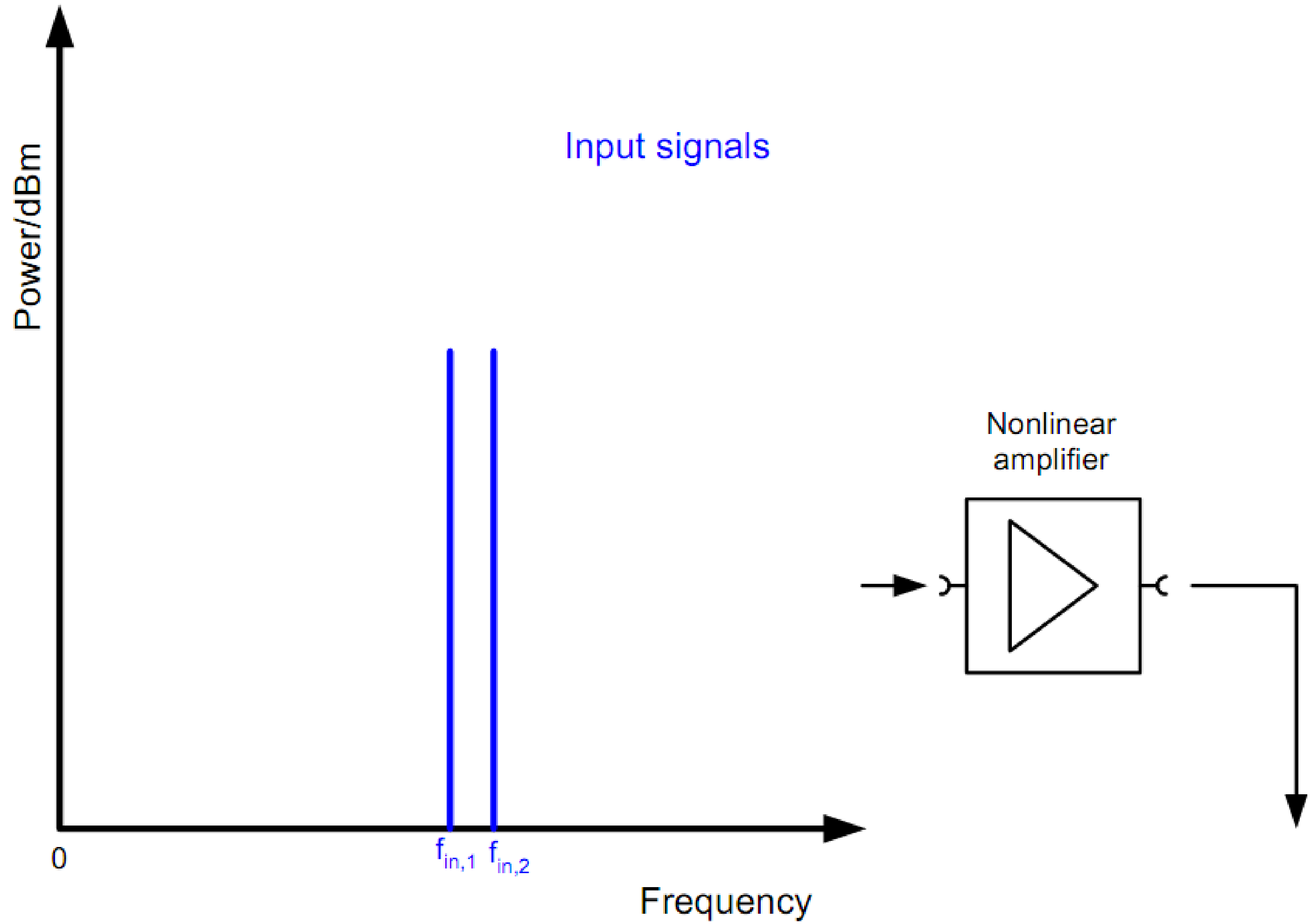
$$\cos(2x)\cdot\sin(y) = \frac{1}{2}\sin(2x - y) + \frac{1}{2}\sin(2x + y)$$

$$\sin(x)\cdot\sin(y) = \frac{1}{2}\cos(x - y) - \frac{1}{2}\cos(x + y)$$

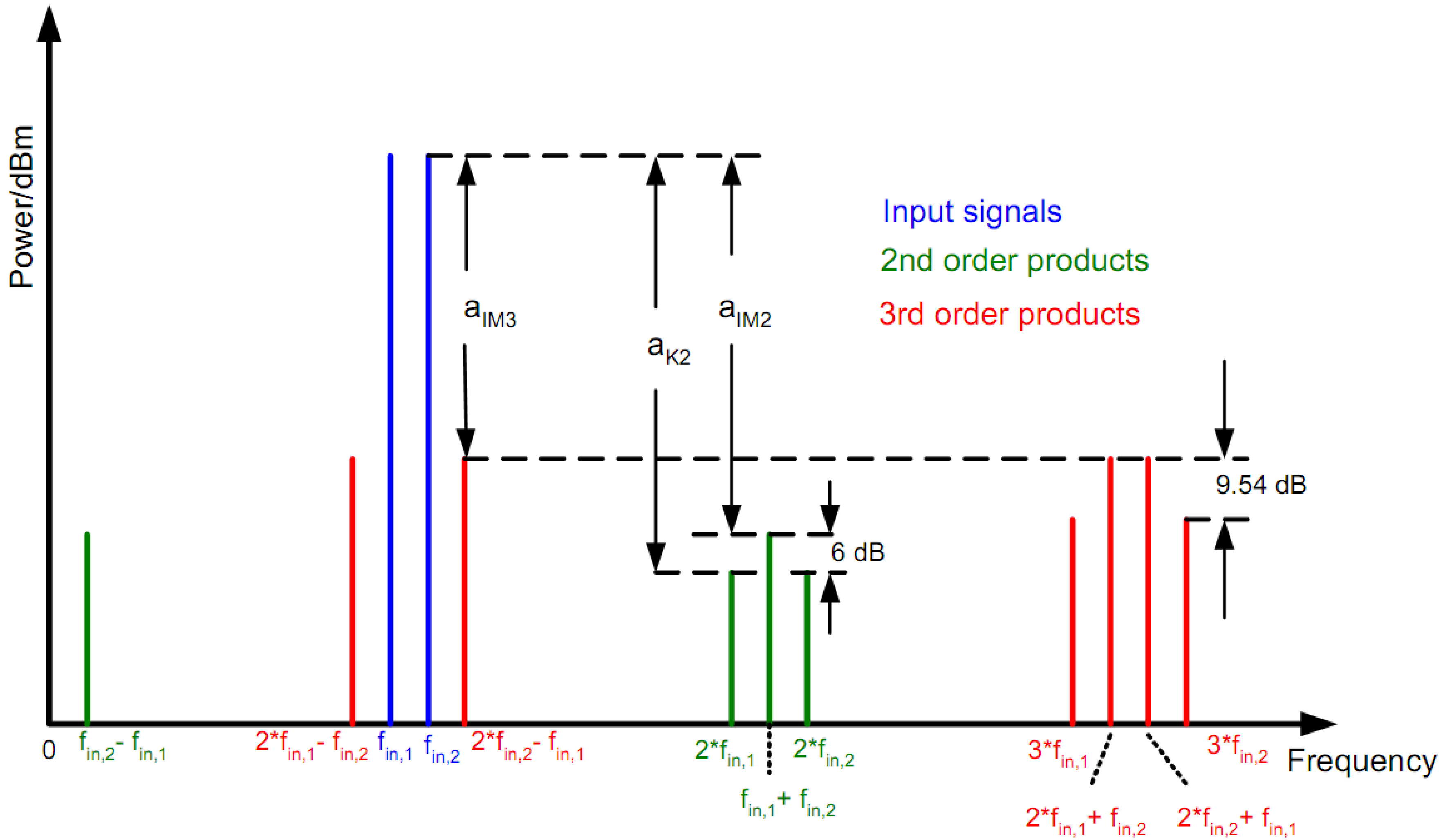
Intermodulation products up to max. 3rd order with two-tone driving

$v_{out}(t) = \frac{1}{2} \cdot a_2 \cdot \left(V_1^2 + V_2^2\right)$	DC component
$+ \left(a_1 \cdot V_1 + \frac{3}{4} \cdot a_3 \cdot V_1^3 + \frac{3}{2} \cdot a_3 \cdot V_1 V_2^2\right) \cdot \sin(\omega_1 \cdot t)$ $+ \left(a_1 \cdot V_2 + \frac{3}{4} \cdot a_3 \cdot V_2^3 + \frac{3}{2} \cdot a_3 \cdot V_1^2 V_2\right) \cdot \sin(\omega_2 \cdot t)$	Fundamental (first harmonic)
$- \frac{1}{2} \cdot a_2 \cdot V_1^2 \cdot \cos(2 \cdot \omega_1 \cdot t)$ $- \frac{1}{2} \cdot a_2 \cdot V_2^2 \cdot \cos(2 \cdot \omega_2 \cdot t)$	Second harmonic
$+ a_2 \cdot V_1 \cdot V_2 \cdot \cos((\omega_2 - \omega_1) \cdot t)$ $- a_2 \cdot V_1 \cdot V_2 \cdot \cos((\omega_2 + \omega_1) \cdot t)$	Second-order intermodulation products
$- \frac{1}{4} \cdot a_3 \cdot V_1^3 \cdot \sin(3 \cdot \omega_1 \cdot t)$ $- \frac{1}{4} \cdot a_3 \cdot V_2^3 \cdot \sin(3 \cdot \omega_2 \cdot t)$	Third harmonic
$+ \frac{3}{4} \cdot a_3 \cdot V_1^2 V_2 \cdot \sin((2 \cdot \omega_1 - \omega_2) \cdot t)$ $- \frac{3}{4} \cdot a_3 \cdot V_1^2 V_2 \cdot \sin((2 \cdot \omega_1 + \omega_2) \cdot t)$ $+ \frac{3}{4} \cdot a_3 \cdot V_1 V_2^2 \cdot \sin((2 \cdot \omega_2 - \omega_1) \cdot t)$ $- \frac{3}{4} \cdot a_3 \cdot V_1 V_2^2 \cdot \sin((\omega_1 + 2 \cdot \omega_2) \cdot t)$	Third-order intermodulation products

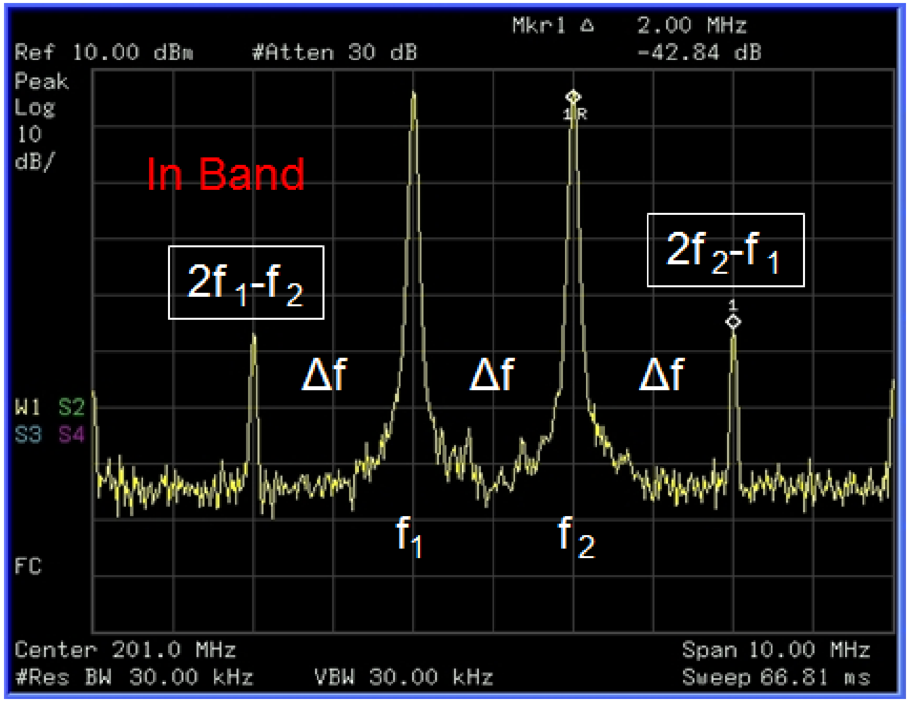
Two-tone driving – Input Signals

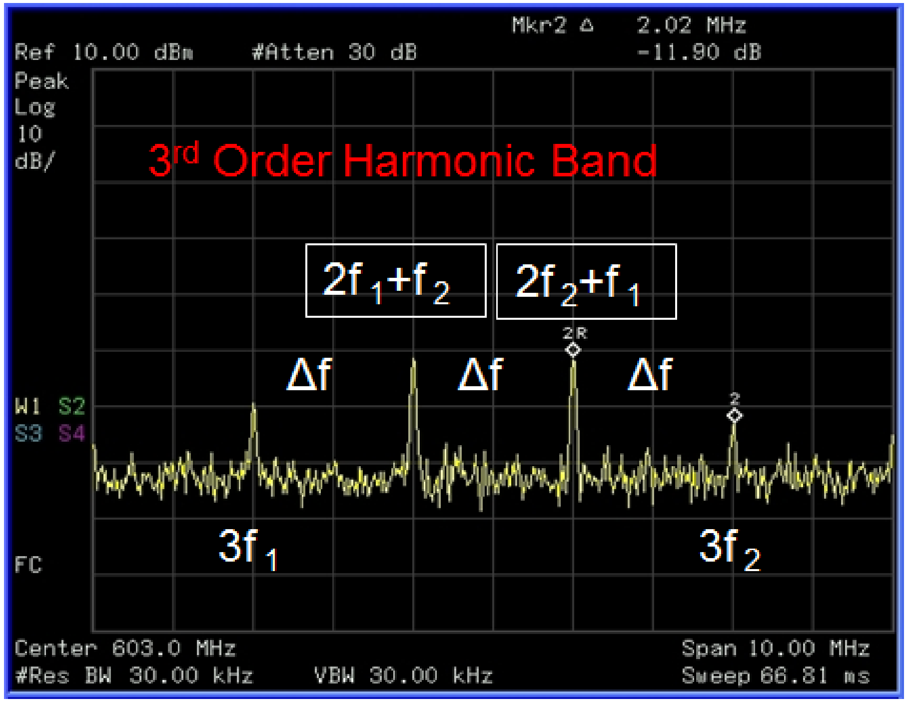


Output spectrum of a nonlinear two-port with two-tone driving for intermodulation products up to max. 3rd order

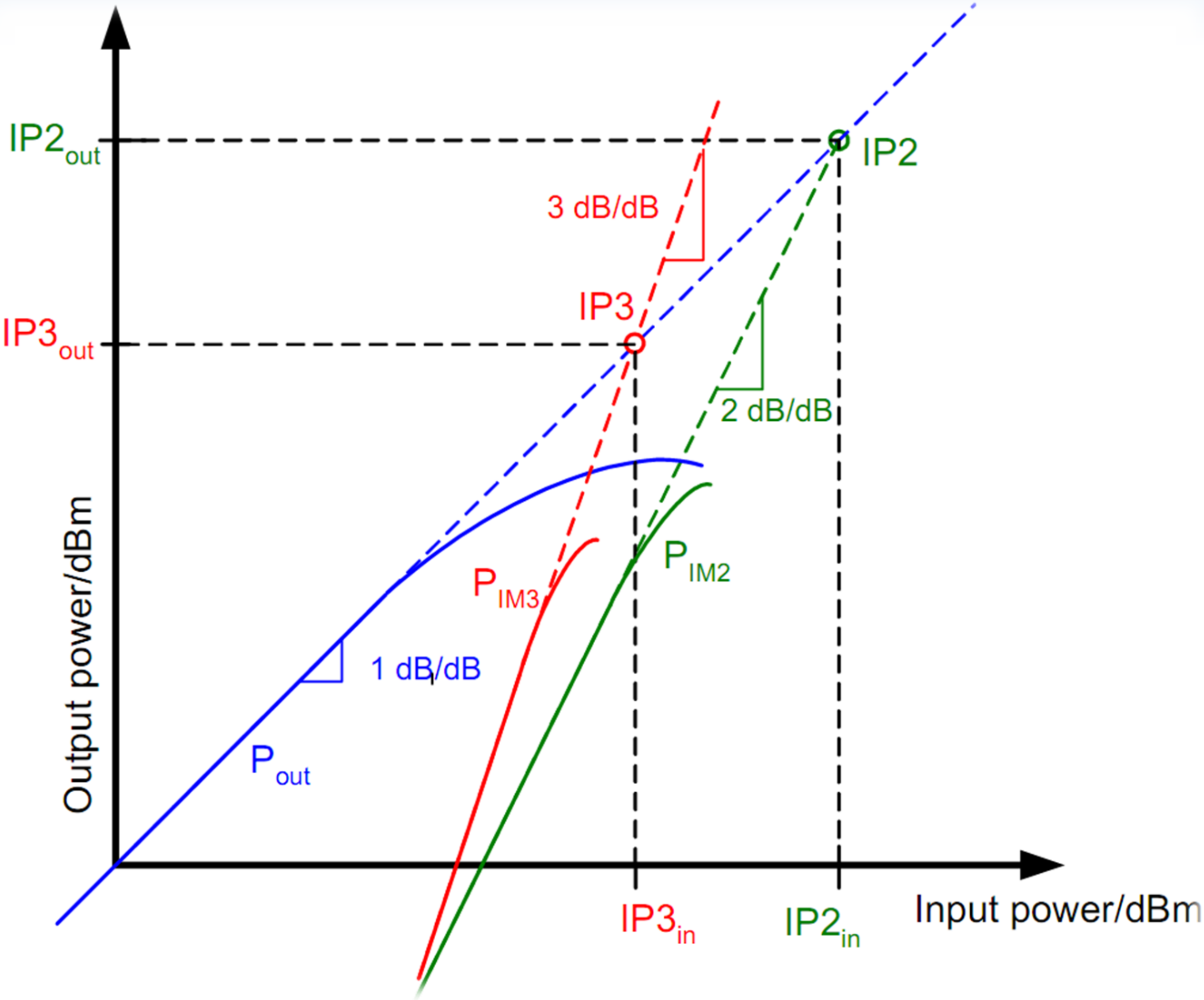


In-Band and Harmonic Band Spectra





Second and Third Order Intercept Points



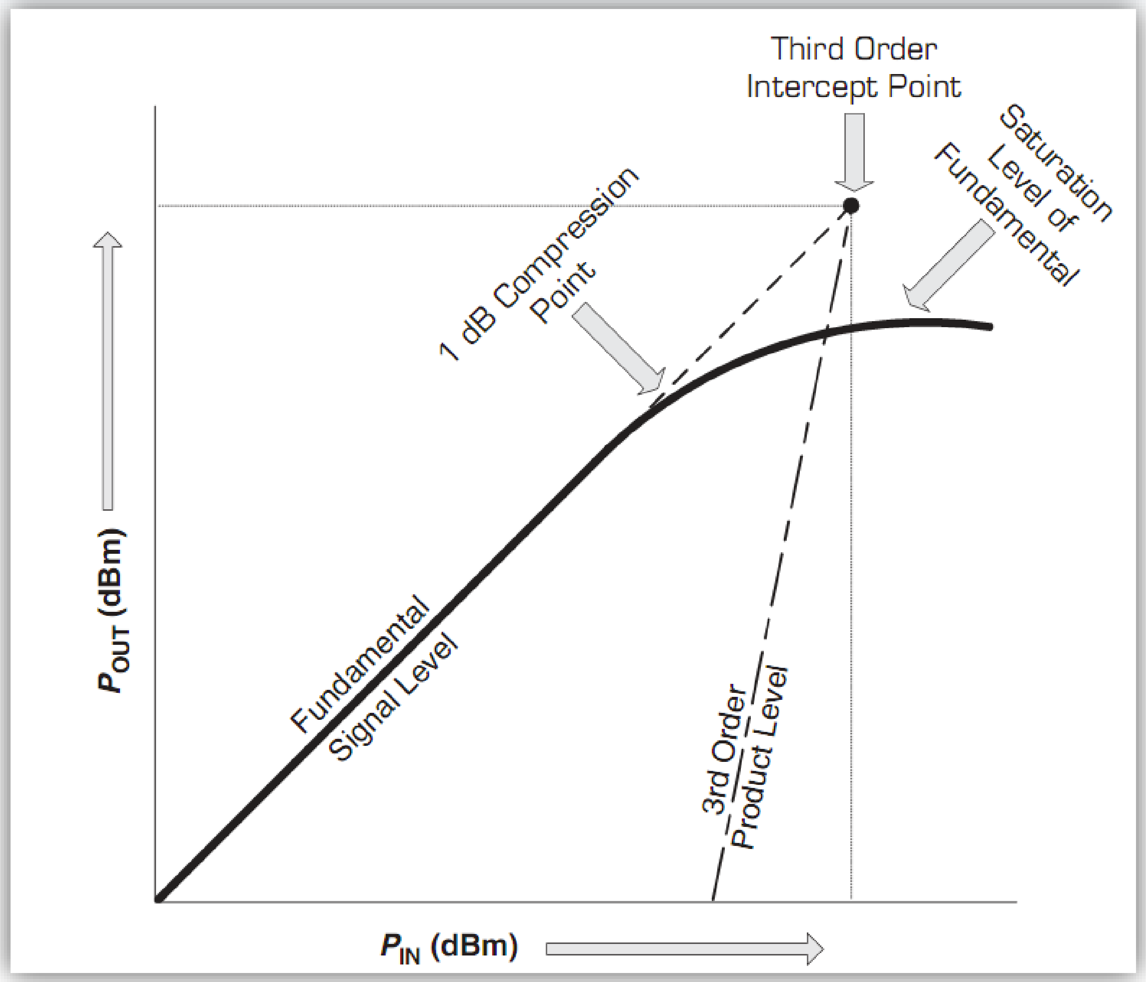
Intermodulation products for $V_1=V_2=V$

$v_{out}(t) = a_2 \cdot V^2$	DC component
$+ \left(a_1 \cdot V + \frac{3}{4} \cdot a_3 \cdot V^3 + \frac{3}{2} \cdot a_3 \cdot V^3\right) \cdot \sin(\omega_1 \cdot t)$	Fundamental (first harmonic)
$+ \left(a_1 \cdot V + \frac{3}{4} \cdot a_3 \cdot V^3 + \frac{3}{2} \cdot a_3 \cdot V^3\right) \cdot \sin(\omega_2 \cdot t)$	
$- \frac{1}{2} \cdot a_2 \cdot V^2 \cdot \cos(2 \cdot \omega_1 \cdot t)$	Second harmonic
$- \frac{1}{2} \cdot a_2 \cdot V^2 \cdot \cos(2 \cdot \omega_2 \cdot t)$	
$+ a_2 \cdot V^2 \cdot \cos((\omega_2 - \omega_1) \cdot t)$	Second-order
$- a_2 \cdot V^2 \cdot \cos((\omega_2 + \omega_1) \cdot t)$	intermodulation products
$- \frac{1}{4} \cdot a_3 \cdot V^3 \cdot \sin(3 \cdot \omega_1 \cdot t)$	Third harmonic
$- \frac{1}{4} \cdot a_3 \cdot V^3 \cdot \sin(3 \cdot \omega_2 \cdot t)$	
$+ \frac{3}{4} \cdot a_3 \cdot V^3 \cdot \sin((2 \cdot \omega_1 - \omega_2) \cdot t)$	Third-order
$- \frac{3}{4} \cdot a_3 \cdot V^3 \cdot \sin((2 \cdot \omega_1 + \omega_2) \cdot t)$	intermodulation products
$+ \frac{3}{4} \cdot a_3 \cdot V^3 \cdot \sin((2 \cdot \omega_2 - \omega_1) \cdot t)$	
$- \frac{3}{4} \cdot a_3 \cdot V^3 \cdot \sin((\omega_1 + 2 \cdot \omega_2) \cdot t)$	

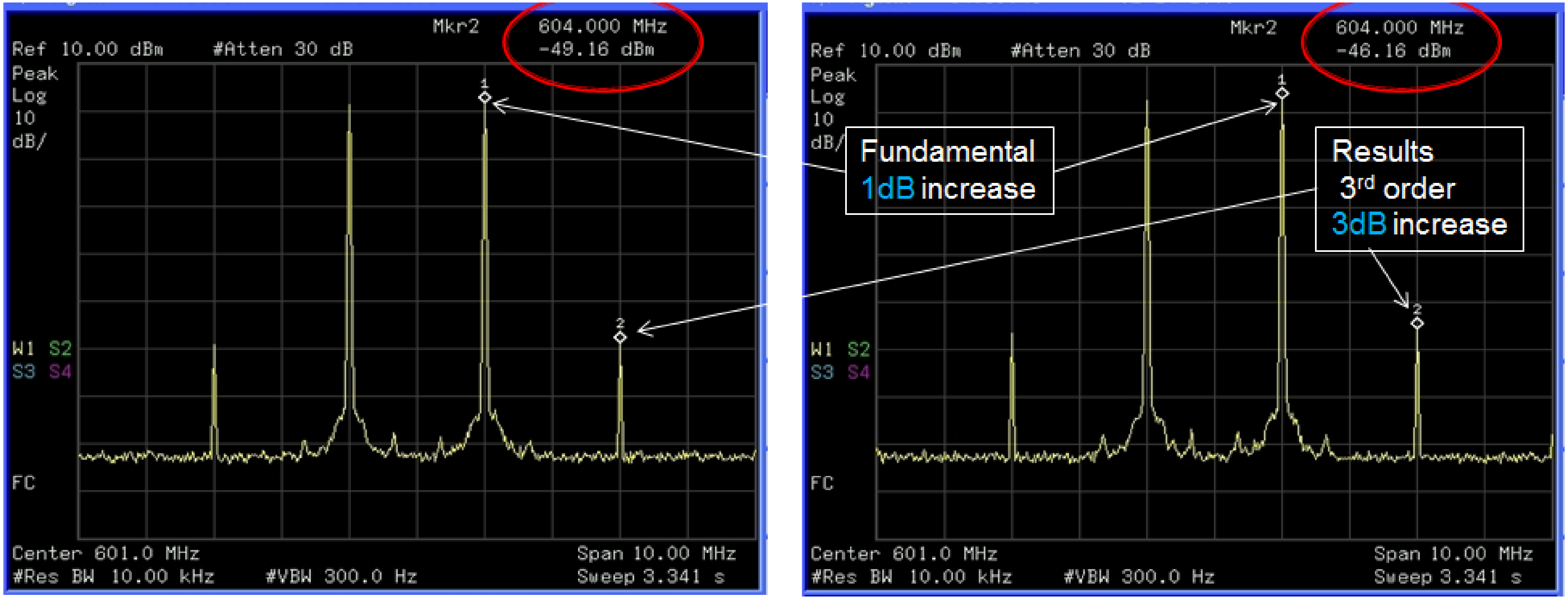
Slope of OIP2 and OIP3 vs. Pin [dBm]

- The log-log power plot of IM_2 is of slope 2dB/dB
- The log-log power plot of IM_3 is of slope 3dB/dB
- The log-log power plot of IM_N is of slope NdB/dB

The third-order intercept and 1 dB compression points



Fundamental vs. 3rd Order



1dB power increase of (each) input signals pair => output 3rd order product increase of 3dB

OIP3 and OIM3 – Linear Scale

$$\text{At IP}_3:\ \frac{a_1^2 A^2}{2} = \frac{9}{4} a_3^2 \left(\frac{A^2}{2}\right)^3$$

$$\text{thus}\quad \left(\frac{A^2}{2}\right)_{IP_3} = \frac{2}{3}\left|\frac{a_1}{a_3}\right|.$$

$$\left(P_{out1}\right)_{IP_3} = \frac{2}{3}\left|\frac{a_1^3}{a_3}\right| = OIP_3.$$

$$\text{Also}\quad IM_{out3} = \frac{9}{4} a_3^2 P_{in}^3 = \left(\frac{3}{2} a_3\right)^2 P_{in}^3 =$$

$$= \left(\frac{3}{2}\frac{a_3}{a_1^3}\right)^2 a_1^6 P_{in}^3 = G^3 \frac{1}{OIP_3^2} P_{in}^3$$

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3rd Order Intermodulation Equations

$$OIP_3[dBm] = IIP_3[dBm] + G[dB]$$

$$P_{in}[dBm] = \Delta P_3[dBc] + Pout_3[dBm] - G[dB]$$

$$\Delta P_3[\mathrm{dBc}] = P_{out1}[\mathrm{dBm}] - P_{out3}[\mathrm{dBm}] =$$

$$= 2(OIP_3[\mathrm{dBm}] - P_{out1}[\mathrm{dBm}]) =$$

$$= \frac{2}{3}(OIP_3[\mathrm{dBm}] - P_{out3}[\mathrm{dBm}])$$

3rd Order Intermodulation Equations (2)

$$OIP_3\,[\text{dBm}] = P_{out1}\,[\text{dBm}] + \frac{\Delta P_3\,[\text{dBc}]}{2}$$

$$P_{out3}\,[\text{dBm}] = 3P_{in}\,[\text{dBm}] + 3G\,[\text{dB}] - 2\,OIP_3\,[\text{dBm}] =$$

$$= 3P_{out1}\,[\text{dBm}] - 2\,OIP_3\,[\text{dBm}]$$

Spurious Free Dynamic Range

- Definition
  - Maximal to minimal input signal power ratio in dB
  - Maximal signal such that the 2-Tone IM products are at the output noise power level
  - Minimal signal equals the sensitivity with a prescribed SNR_{out}.

    Assume here SNR_{out}=1 (0dB).

Spurious Free Dynamic Range (cont'd)

$$\Delta P_3[\text{dBc}] = \frac{2}{3}(OIP_3[\text{dBm}] - P_{out3}[\text{dBm}])$$

For 3rd order IM Products at the noise level:

$$DR = \frac{2}{3}[OIP_3 - 10\log(kT_{in}BF_rG)][dB]$$

If $SNR_{out} \neq 0$dB in the sensitivity definition, then:

$$DR = \frac{2}{3}[OIP_3 - 10\log(kT_{in}BF_rG(\frac{S}{N})_{out})][dB]$$

Cascade Intercept Point

- Assuming incoherent combining of IM products it is possible to show that:

2nd Order IM's:

$$\frac{1}{OIP_2^{sys}} = \frac{1}{G_2 \ldots G_N \cdot OIP_2^{(1)}} + \frac{1}{G_3 \ldots G_N \cdot OIP_2^{(2)}} + \ldots + \frac{1}{G_N \cdot OIP_2^{(N-1)}} + \frac{1}{OIP_2^{(N)}}$$

3rd Order IM's:

$$\frac{1}{(OIP_3^{sys})^2} = \frac{1}{(G_2 \ldots G_N \cdot OIP_3^{(1)})^2} + \frac{1}{(G_3 \ldots G_N \cdot OIP_3^{(2)})^2} + \ldots + \frac{1}{(G_N \cdot OIP_3^{(N-1)})^2} + \frac{1}{(OIP_3^{(N)})^2}$$

Cascade Intercept Point – Another Form

- Assuming incoherent combining of IM products it is possible to show that:

2nd Order IM's:

$$\frac{1}{OIP_2^{sys}} = \frac{G_T}{G_1 OIP_2^{(1)}} + \frac{G_T}{G_1 G_2 OIP_2^{(2)}} + \ldots + \frac{G_T}{G_T OIP_2^{(N)}}$$

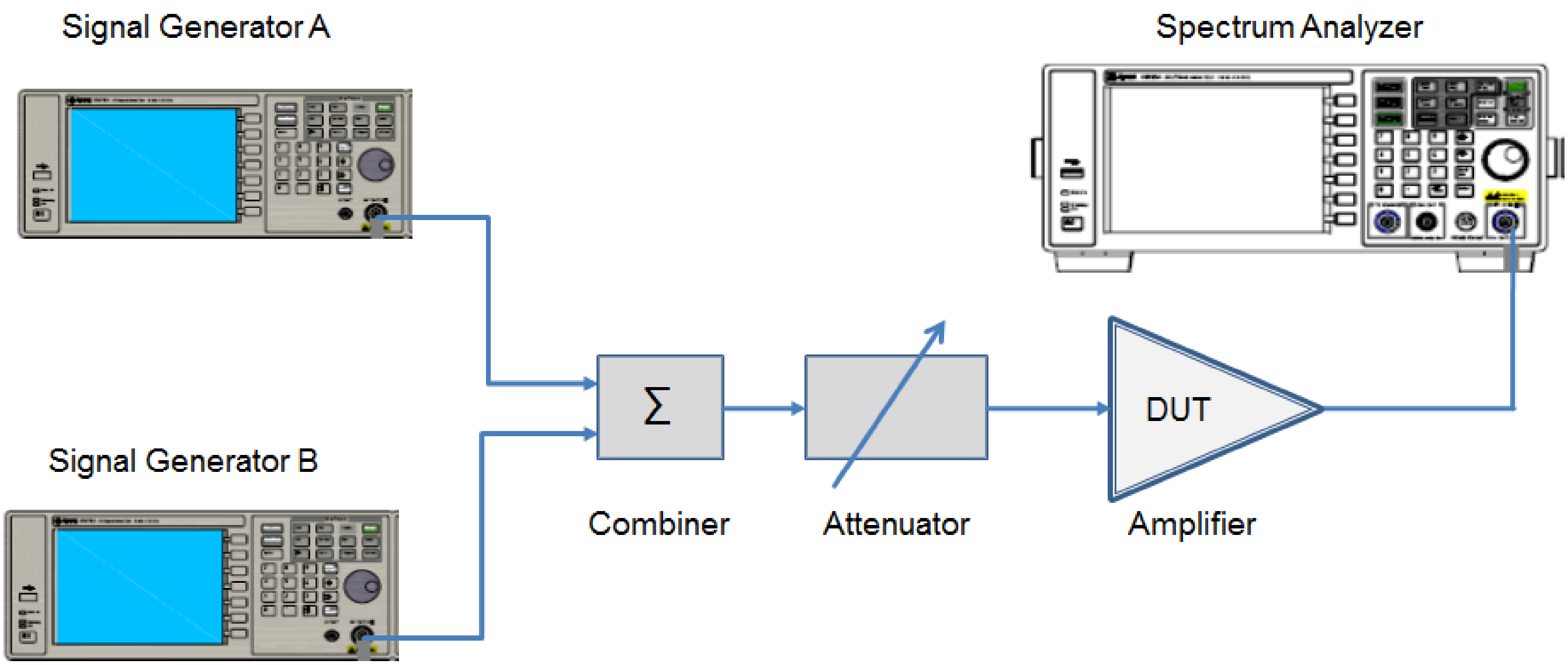
3rd Order IM's:

$$\frac{1}{(OIP_3^{sys})^2} = \frac{(G_T)^2}{(G_1 OIP_3^{(1)})^2} + \frac{(G_T)^2}{(G_1 G_2 OIP_3^{(2)})^2} + \ldots + \frac{(G_T)^2}{(G_T OIP_3^{(N)})^2}$$

Measuring Nonlinear Behavior

- Most common measurements: Second level
  - using a network analyzer and power sweeps
    - gain compression
    - AM to PM conversion
  - using a spectrum analyzer + source(s)
    - harmonics, particularly second and third
    - intermodulation products resulting from two or more RF carriers

Two Tone Test – Setup



Third order Spurious Free Dynamic Range, SFRD-3

- Spurious Free Dynamic Range
- Definition
  - Maximal to minimal input signal power ratio in dB
  - Maximal signal such that the 2-Tone IM products are at the output noise power level
  - Minimal signal equals the sensitivity with a prescribed SNR_{out}.
    Assume here (or if not specified otherwise) $SNR_{out}=1$ (0dB).

Design Tradeoffs between linearity and Sensitivity Optimization

- Sensitivity Optimization
  - First stage with high gain
  - First stage with low NF

- Linearity Optimization
  - Limit the gain of the first stages
  - Last stage with high IP